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Pion Cloud Contributions to the Proton's Weak Magnetic Form Factor

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Abstract

We report on the contributions to the nucleon weak magnetic and electric form factors arising from isospin-breaking form factors. The relativized quark model used in the calculations includes isospin impurities in the large basis nucleon wavefunctions and uses constituent quark currents with both Pauli and Dirac components. Significant contributions are found to arise from the Pauli component when it is interpreted as arising from pion-baryon loops. Including these pion cloud contributions reduces the strange magnetic form factor extracted from the recent SAMPLE measurement of the proton's weak neutral magnetic form factor at low Q^2 by 35 to 52%.

There has been considerable interest for several years now in the possibility of measuring the effects of the strange quark-antiquark ($\overline{s}s$) pair in the nucleon using the parity-violating asymmetry in elastic electron scattering from the nucleon [1]. This year the first measurement of this asymmetry was reported on by the SAMPLE Collaboration [2] and clearly demonstrates the feasibility of seeing asymmetries of a few parts per million. The experiment was performed at the MIT/Bates Linear Accelerator Center using 200 MeV incident electrons elastically scattered from the proton at backward angles with an average Q^2 of 0.1 (GeV/c)^2. With this choice of kinematics the neutral weak form factor for the proton G_M^Z is enhanced relative to the corresponding electric term and can be extracted from the asymmetry using the relation

$$A = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = -\frac{G_F Q^2}{\pi\alpha\sqrt{2}} \times \left\{ \frac{\epsilon G_E^\gamma G_E^Z + \tau G_M^\gamma G_M^Z - \frac{1}{2}(1-4\sin^2\theta_W)\epsilon' G_M^\gamma G_A^Z}{\epsilon (G_E^\gamma)^2 + \tau (G_M^\gamma)^2} \right\}, \tag{1}$$

where ϵ, τ, and $\epsilon' = \sqrt{\tau(1+\tau)(1-\epsilon^2)}$ are kinematic quantities and $Q^2 > 0$ is the four momentum transfer squared. The electromagnetic electric and magnetic form factors for the proton are denoted by G_E^γ and G_M^γ respectively. The corresponding electric and magnetic neutral weak form factors are denoted by the superscript Z. The numerator also contains the neutral weak axial-vector form factor G_A^Z which causes a significant correction in the extracted value of G_M^Z.

The interest in the neutral weak magnetic form factor lies in the simple relationship it has to the strange magnetic form factor G_M^S when certain simplifying assumptions are made. To tree level order, and assuming no isospin breaking in the nucleon, one finds using the notation of Ref. [2]

$$G_M^Z = \frac{1}{4}(G_M^p - G_M^n) - \sin^2\theta_W G_M^p - \frac{1}{4} G_M^S, \tag{2}$$

where θ_W is the weak mixing angle known from a recent high precision measurement [3]: $\sin^2\theta_W(M_Z) = 0.2315 \pm 0.0004$. Electroweak radiative corrections have been applied to Eq. (2) in the report on the SAMPLE experiment. However, the effects of isospin breaking are not considered in their analysis.

In this Letter we report on a detailed calculation of the additional form factors arising from isospin breaking and follow the notation of Ref. [4]. The isoscalar $(u+d)$ and isovector $(u-d)$ Dirac (F_1) and Pauli (F_2) form factors for the proton and neutron are defined by

$$\begin{aligned}
&\langle N(p')|\frac{1}{2}\left(\overline{u}\gamma_\mu u-\overline{d}\gamma_\mu d\right)|N(p)\rangle \\
&\qquad\equiv\overline{\mathcal{U}}(p')\left[\frac{1}{2}\left({}^{u-d}F_1^{p+n}\pm{}^{u-d}F_1^{p-n}\right)\gamma_\mu+\frac{1}{2}\left({}^{u-d}F_2^{p+n}\pm{}^{u-d}F_2^{p-n}\right)i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}\right]\mathcal{U}(p) \\
&\langle N(p')|\frac{1}{6}\left(\overline{u}\gamma_\mu u+\overline{d}\gamma_\mu d\right)|N(p)\rangle \\
&\qquad\equiv\overline{\mathcal{U}}(p')\left[\frac{1}{2}\left({}^{u+d}F_1^{p+n}\pm{}^{u+d}F_1^{p-n}\right)\gamma_\mu+\frac{1}{2}\left({}^{u+d}F_2^{p+n}\pm{}^{u+d}F_2^{p-n}\right)i\sigma_{\mu\nu}\frac{q^\nu}{2M_N}\right]\mathcal{U}(p), \quad (3)
\end{aligned}$$

where the sign is + for the proton and − for the neutron. The corresponding Sachs form factors are given by the linear combinations $G_E = F_1 - \tau F_2$ and $G_M = F_1 + F_2$. For the neutral weak magnetic form factor of the proton Eq. (2) is modified by replacing the strange magnetic form factor G_M^S by the linear combination $G_M^S +{}^{u+d} G_M^{p-n} -{}^{u-d} G_M^{p+n}$. Note that the strange form factor arises from the current operator $\overline{s}\gamma_\mu s$ whereas the isospin-breaking terms arise from isovector and isoscalar currents with factors of $\frac{1}{2}$ and $\frac{1}{6}$ respectively.

The calculations of the form factors ${}^{u+d}G_{E,M}^{p-n}$ and ${}^{u-d}G_{E,M}^{p+n}$ utilize the relativized quark model based on the light-cone formalism of Ref. [5] which produces realistic mixed wavefunctions for each nucleon by solving the three-body problem in a large basis of harmonic oscillator states. The model allows for the constituent quarks to have (a) different masses for the u and d quarks, (b) electromagnetic interactions (Coulomb + hyperfine) between the quarks, and (c) both Dirac and Pauli electromagnetic form factors corresponding to the substitution

$$\overline{q}e_q\gamma_\mu q \longrightarrow \overline{q}_c e_q\gamma_\mu f_{1q}q_c + \overline{q}_c\kappa_q^{(N)} i\sigma_{\mu\nu}\frac{q^\nu}{2m_q}f_{2q}q_c, \quad (4)$$

where q_c denotes a constituent quark appropriate to the model. In the u_c, d_c sector the Dirac component can be written as isoscalar and isovector currents for each nucleon as in Eqs. (3). The Pauli component term involves anomalous moments $\kappa_q^{(N)}$ which depend on the quark flavor *and* the nucleon flavor. This is an important assumption which is understandable

if one assumes that the Pauli term arises from interactions with the charged pion cloud corresponding to the $\overline{u}u$ and $\overline{d}d$ loops in Fig. 1.

In these calculations there are isospin-breaking effects associated with both of the quark form factors. The Dirac quark component has isospin violations because of the charge dependence of the Hamiltonian used. In the expansion of the proton and neutron wavefunctions into a basis of states with good isospin given by

$$\Psi_N^{J^P=\frac{1}{2}^+} = \sum_\alpha a_\alpha(N)\phi_\alpha^{\frac{1}{2}^+}(T=\frac{1}{2},T_z) + \sum_\beta b_\beta(N)\phi_\beta^{\frac{1}{2}^+}(T=\frac{3}{2},T_z), \tag{5}$$

there are mirror violations from the fact that $a_\alpha(p)$ is slightly different from $a_\alpha(n)$ as well as the usual violations *via* the admixture of the $T=\frac{3}{2}$ delta configurations. The former $T=\frac{1}{2}$ 'dynamic distortion' is larger than the $T=\frac{3}{2}$ violation and both contribute to the isospin-breaking form factors calculated here. The rms averages when 95 basis states (corresponding to oscillator quanta up to $8\hbar\omega$) are used are $A = (\sum_\alpha [a_\alpha(p) - a_\alpha(n)]^2)^{\frac{1}{2}} = 1.57 \times 10^{-3}$ and $B = (\sum_\beta [b_\beta(p) + b_\beta(n)]^2)^{\frac{1}{2}} = 0.88 \times 10^{-3}$, yielding a total average violation $C = (A^2 + B^2)^{\frac{1}{2}} = 1.8 \times 10^{-3}$. The isospin-breaking form factors ${}^{u+d}G^{p-n}$ and ${}^{u-d}G^{p+n}$ in our calculations do show contributions from the Dirac quark component which are of the order of magnitude of C.

Violations arising from the Pauli quark component are of considerable interest because they are entirely analogous to one of the mechanisms proposed [6] for the strange form factors of the nucleon. In the case of $\overline{s}s$ loops the $\overline{s}$ coalesces with a u quark to form a K^+ and the s joins with the spectator pair of quarks to form a Λ or Σ baryon. The difference in the momentum distributions of an $\overline{s}$ in the K^+ and an s in the hyperons causes the strange form factors to be non-zero. Such non-perturbative effects from the loops involving the lightest meson and baryons are expected at low Q^2. By analogy we see in Fig. 1 that the pion-nucleon and pion-delta loops should have strong differences in the momentum distributions for the $\overline{u}$, $\overline{d}$ in the pion and their u, d partners in the nucleon or delta. In our parameterization of the anomalous moments we assume the photon or Z interacts predominantly with the pion. This is expected to be a valid assumption because for the light baryons the pion is in a p

orbit and with its small mass it will dominate the quark Pauli component contributions to both the nucleonic charge radius and the nucleonic anomalous magnetic moment. We also require the relations $\kappa_u^{(p)} = -\kappa_d^{(n)}$ and $\kappa_d^{(p)} = -\kappa_u^{(n)}$ due to charge symmetry which relates π^+ to π^- and u to d when transforming the proton into a neutron by the charge symmetry operator, *i.e.* a 180^o rotation about the y axis in isospin space. These relations can be visualized by noting the similar hadron content of the *left* diagrams in Fig. 1, which in our model give $\kappa_u^{(p)}$ and $\kappa_d^{(n)}$, and noting the sign of the photon or Z coupling to the pion in the loop, and similarly for the *right* diagrams. Note that the right diagrams will be suppressed due to the lack of a nucleon–pion intermediate state. Previous calculations using Pauli quark components [7] appear to have ignored the importance of charge symmetry which relates constituent u quarks in the proton to constituent d quarks in the neutron and *vice-versa*.

The parameters for the quark-model Hamiltonian are determined by fitting the baryon spectrum, and the quark mass difference $m_d - m_u = 3.6$ MeV is fixed by fitting the nucleon mass difference $M_n - M_p = 1.29$ MeV. The parameters of the current quark operators $\kappa_u^{(p)}$, $\kappa_d^{(p)}$, and the form factors $f_i(Q^2) = (1 + Q^2/\Lambda_i^2)^{-i}$ with $i = 1, 2$ are determined by fitting the observed proton and neutron electric and magnetic form factors. These fits deviate from experiment by amounts which are typically of the order of 5% for Q^2 up to 1 (GeV/c)^2, except for the neutron form factors which are not known to this accuracy. The values of $\kappa_u^{(p)}$, $\kappa_d^{(p)}$, and the Λ_i are predominantly determined by the nucleon magnetic moments and the neutron rms charge radius. Without the pion cloud contributions contained in the Pauli quark current the magnitude of the nucleon magnetic moments are underpredicted by about 0.6 n.m. Similarly we find that the pion cloud provides about 50% of the neutron charge radius. Using the parameters $\kappa_u^{(p)} = 0.12 = -\kappa_d^{(n)}$, $\kappa_d^{(p)} = -0.04 = -\kappa_u^{(n)}$, and $\Lambda_1^2 = 1.22$ GeV^2, $\Lambda_2^2 = 0.30$ GeV^2, we obtain the $u + d$ isospin-breaking form factors shown in Figure 2.

We have not plotted the corresponding $u - d$ terms in Fig. 2 since, as shown in Table I, their numerical values are too small to be observed. The numerical results in Table I also include the form factor calculations for the Dirac current alone (all $\kappa_q^{(N)} = 0$) and show that the large values of the $u + d$ terms arise almost entirely from the pion-cloud contributions

to the Pauli term. The isovector nature of the pion cloud arises because we have assumed that the contribution from the quark partner in the $\overline{q}q$ loops of Fig. 1 are suppressed relative to the antiquark partner residing in the pion. If one assumed that this quark contribution was the same as that of its antiquark partner then the anomalous quark moments would be proportional to the constituent quark charges and the same for each flavor in the neutron and proton, *i.e.* the Pauli term would be like the Dirac term and yield only small $u + d$ isospin-breaking form factors (of the order of $C = 2 \times 10^{-3}$) for the nucleon. We contend that the dynamical symmetry breaking mechanism which places the antiquark in a Goldstone boson (small mass) and its quark partner in a nucleon or delta baryon will produce Pauli quark form factors which have anomalous moments of the type used here. The ratio of the u and d-quark anomalous moments arising from the diagrams in Fig. 1 have been estimated by us to be approximately -2 to -3. Calculations with a ratio of -2 reduce the magnitude of the form factors in Fig. 2 by about 30%. At $Q^2 = 0.1$ (GeV/c)^2 we obtain a contribution to the weak neutral form factor from the pion cloud of between -0.02 and -0.03 n.m. Inclusion of this contribution in the SAMPLE collaboration results would reduce the extracted value of the strange magnetic form factor at this Q^2 (*i.e.* $G_M^S = +0.23 \pm 0.09 \pm 0.04 \pm 0.05$) by 35 to 52%.

In summary we believe the pion cloud effects calculated here are comparable in size to the kaon cloud effects expected for the corresponding strange form factors for the nucleon. We expect the Q^2 dependences of the pion and kaon cloud contributions to be different due to the significant difference in their masses. It will be interesting to see if the planned experiments [1] can provide information on both the pion and kaon cloud components of the nucleon and ^{4}He. Considerations of other sources of isospin breaking and strange form factors [8], such as photon coupling to the vector mesons ρ, ω, ϕ, *etc.*, will be needed. In particular the effects of both $\rho - \omega$ mixing on ${}^{u\pm d}G_{E,M}^{p\mp n}$ and of $\omega - \phi$ mixing on $G_{E,M}^S$ need to be better understood. A more detailed account of this work including applications to ^{4}He will be reported elsewhere.

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TABLES

TABLE I. Numerical values of the isospin-breaking form factors ${}^{u+d}G_{E,M}^{p-n}$ and ${}^{u-d}G_{E,M}^{p+n}$ for $0.1 \leq Q^2 \leq 0.8$ GeV^2 for (a) all $\kappa_q^{(N)} = 0$ and (b) $\kappa_u^{(p)} = 0.12 = -\kappa_d^{(n)}$, and $\kappa_d^{(p)} = -0.04 = -\kappa_u^{(n)}$ as described in the text. Units of G_E are 10^{-3}, and units of G_M are 10^{-3} n.m.

	(a)				(b)			
Q^2 (GeV/c)^2	${}^{u+d}G_E^{p-n}$	${}^{u+d}G_M^{p-n}$	${}^{u-d}G_E^{p+n}$	${}^{u-d}G_M^{p+n}$	${}^{u+d}G_E^{p-n}$	${}^{u+d}G_M^{p-n}$	${}^{u-d}G_E^{p+n}$	${}^{u-d}G_M^{p+n}$
0.0	0.0	3.0	0.0	1.3	0	250	0	3.7
0.1	0.0	2.3	−1.5	2.5	−27	124	-2.1	3.5
0.2	−0.1	1.6	−2.2	2.0	−30	56	-2.8	2.3
0.3	−0.2	1.2	−2.6	1.4	−28	31	-3.2	1.5
0.4	−0.2	0.8	−2.7	0.9	−25	18	-3.2	0.9
0.5	−0.2	0.6	−2.8	0.5	−23	11	-3.2	0.4
0.6	−0.2	0.4	−2.7	0.2	−20	6	-3.0	0.1
0.7	−0.2	0.3	−2.5	−0.1	−18	3	-2.9	−0.1
0.8	−0.2	0.2	−2.4	−0.2	−15	2	-2.7	−0.3

FIGURES

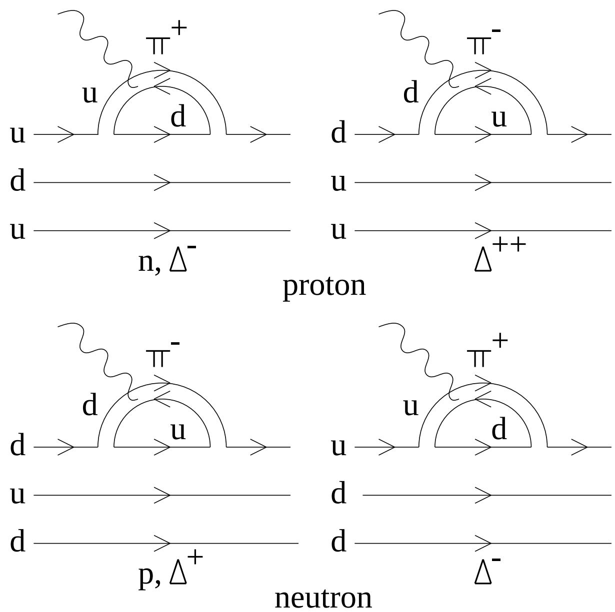

FIG. 1. Virtual $\overline{q}q$ pairs which contribute to the Pauli component for constituent quarks in each nucleon.

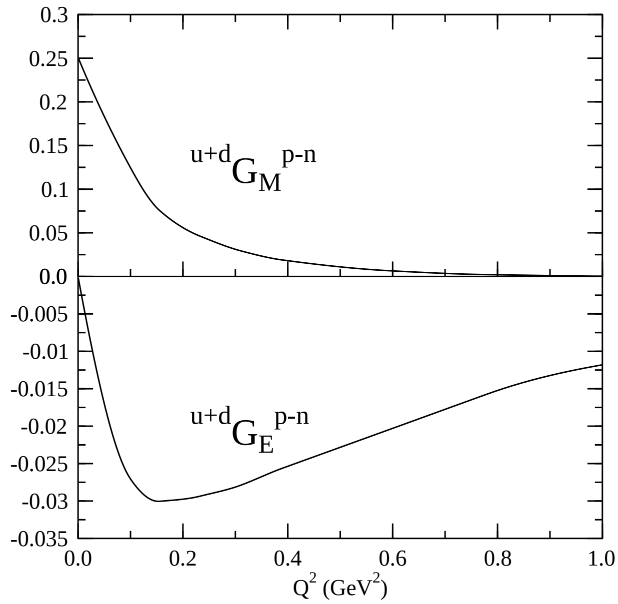

FIG. 2. Predictions for the isospin-breaking form factors ${}^{u+d}G_E^{p-n}$, ${}^{u+d}G_M^{p-n}$ which contribute to the neutral weak electric and magnetic proton form factors respectively. Note the different vertical scales above and below the Q^2 axis.